

Computationally Efficient Signal Detection with Unknown Bandwidths

Ali Rasteh, Sundeep Rangan

Abstract—Signal detection in environments with unknown signal bandwidth and time intervals is a basic problem in adversarial and spectrum-sharing scenarios. This paper addresses the problem of detecting signals occupying unknown degrees of freedom from non-coherent power measurements where the signal is constrained to an interval in one dimension or a hypercube in multiple dimensions. A Generalized Likelihood Ratio Test (GLRT) is derived, resulting in a straightforward metric involving normalized average signal energy on each candidate signal set. We present bounds on false alarm and missed detection probabilities, demonstrating their dependence on signal-to-noise ratios (SNR) and signal set sizes. To overcome the inherent computational complexity of exhaustive searches, we propose a computationally efficient binary search method, reducing the complexity from $O(N^2)$ to $O(N)$ for one-dimensional cases. Simulations indicate that the method maintains performance near exhaustive searches and achieves asymptotic consistency, with interval-of-overlap converging to one under constant SNR as measurement size increases. The simulation studies also demonstrate superior performance and reduced complexity compared to contemporary neural network-based approaches, specifically outperforming custom-trained U-Net models in spectrum detection tasks.

Index Terms—Spectrum Sensing, Efficient Spectrum Detection, Cognitive Radio, Maximum Likelihood Estimation (MLE), Binary Hypothesis Testing, Neural Networks for Signal Detection

I. INTRODUCTION

SIGNAL detection is a fundamental problem in various fields such as communications, radar, and biomedical engineering [1]–[4]. In many problems, the time interval and bandwidth that the signal occupies are not known *a priori*. In these cases, signal detection must be performed in parallel with time and bandwidth estimation. This situation arises most obviously in adversarial scenarios where an interfering signal can have an arbitrary center frequency, bandwidth, and time interval. The situation may also arise in future spectrum co-existence scenarios where multiple services use arbitrary bandwidths or frequency hopping for flexibility [2], [5]–[7]. Signal detection is also potentially important in emerging systems in the upper mid-band for co-existence between disparate systems [8], [9].

In this paper, we consider the general problem of detecting a signal from a set of non-coherent power measurements. The signal, when present, occupies an unknown set S of the degrees of freedom. One power measurement is made on each

degree of freedom, and the set of power measurements are modeled exponential random variables with a higher mean on the degrees of freedom in S . We consider both one-dimensional and multi-dimensional search problems, where the signal set S consists of an unknown interval or hypercube within the measurement space. The dimensions of the measurement space could be frequency, time, angle of arrival, etc.

Our contributions are as follows.

- *Derivation of the GLRT*: We derive a simple generalized likelihood ratio test (GLRT) [10], [11] for the detection problem. For each possible candidate signal set S that the signal can occupy, it is shown that the GLRT can be computed from a simple metric of the average signal energy on the interval S normalized by the number of degrees of freedom in S . The GLRT then maximizes the metric over the sets.
- *Receiver Operating Characteristic*: Bounds are provided for both the false alarm and missed detection probability, with the missed detection probability depending on the size of the signal set and the SNR.
- *Computationally efficient binary search*: A key challenge in the GLRT is that all signal sets S must be searched. For a one-dimensional problem with N degrees of freedom, the search over all intervals will be of complexity $O(N^2)$. We present a binary search method with a complexity of $O(N)$.
- *Asymptotic consistency of binary search*: Our main theoretical result (Theorem 4) shows that under constant SNR per degree of freedom, the interval-of-overlap goes to one almost surely as the block size N to infinity.
- *Simulations performance*: We conduct a number of simulations and show that the computationally efficient binary search method performs similarly to the exhaustive search. In addition, the binary search is asymptotically consistent as the interval length grows.
- *Comparison with neural network approaches*: Given the success of neural networks in image segmentation, considerable work has focused on their application in spectral detection [12]–[17]. We show that the binary search method outperforms custom-trained U-Net neural networks with considerably less complexity.

Related Works

A comprehensive survey of various spectrum sensing methodologies can be found in [1]–[4]. The methods in this paper can be considered as a type of energy detection method

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[18]–[21] which is most useful when limited signal information is available. When additional information is available, several other techniques can be used to improve detection [1]. For example, one can use classical matched filtering if the transmitted signal is known [22], [23]; as well as feature detection methods [24]–[26] and eigenvalue techniques [27] that can exploit other signal properties.

For frequency domain processing, many signal detectors operate on transformed data such as FFTs [28], wavelets [29], [30], and filter banks [31], [32]. Our work is closest in style to [28] that also uses frequency-domain data. The key difference is that [28] assumes some band structure that is known a priori, such as a WiFi signal being on one of a known set of bands. In contrast, the key challenge in the present work is that the bandwidth is not known a priori and can occupy an arbitrary interval. It should be recognized that most of these transformed-based methods are wideband, meaning the detection data is at a higher sample rate than the true signal. Several sub-Nyquist methods have also been explored via compressive sensing [33]–[35] and multi-coset sensing [36]. However, in this work, we do not attempt any super-resolution of frequency or time.

Cabric et al. [37], [38] experimentally evaluated spectrum sensing methods, focusing on trade-offs and scaling laws for sensing time in AWGN channels. One key result is that the number of samples required for non-coherent detection methods (such as the one in this paper) grows as $1/\text{SNR}^2$ in comparison to $1/\text{SNR}$ for coherent match filtering. While we do not provide scaling laws for the SNR in this problem, our results show that we obtain a consistent estimation with constant SNR per bin and growing signal length.

Numerous studies have endeavored to enhance the energy efficiency of the spectrum detection problem; however, these efforts have predominantly concentrated on energy usage of the communication, rather than the performance of detection algorithms [39]–[43]. The works that have considered detection complexity have mostly focused on reducing the complexity of deep learning methods [44], [45] and compressive sensing [46]. In our simulations section, we will compare against a custom deep learning method and show that, even at high complexity, the proposed method offers better performance.

II. PROBLEM FORMULATION

A. Linear Search Problem

For simplicity, we first consider a linear search problem. We are given N frequency bins indexed $n = 0, \dots, N-1$ and a vector of power measurements $\mathbf{X} = (X_0, \dots, X_{N-1})$. A signal occupies an unknown interval $S = [a, b]$ where $0 \leq a \leq b \leq N$ – see Fig. 1(a). Note that a may equal b so that S is empty. Given the interval S , the received power measurements are modeled as exponential and independent with expectation

$$\mathbb{E}[X_n] = \begin{cases} 1 + \gamma & \text{if } n \in S \\ 1 & \text{if } n \notin S, \end{cases} \quad (1)$$

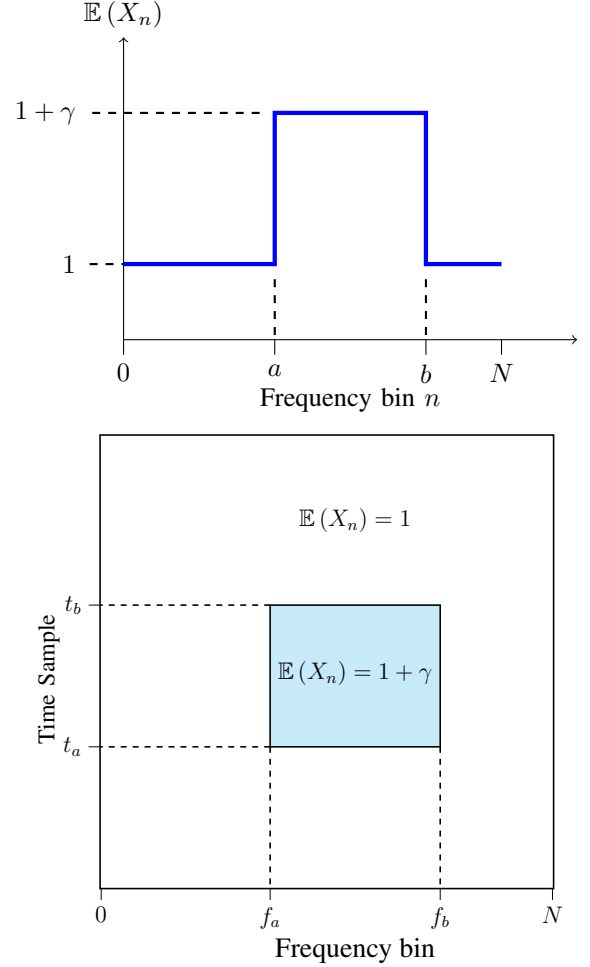


Fig. 1: Detection problem examples: Top: $d = 1$ example for finding an interval $[a, b]$ of signal energy in N frequency bins; Bottom: $d = 2$ example of finding a bounding box in time-frequency.

where γ represents an SNR. Here, we have normalized the power values so that $\mathbb{E}(X_n) = 1$ corresponds to the case of a noise only signal.

Our problem is twofold: First, we wish to determine if a signal is present. We model this first problem as a hypothesis testing problem. We denote the null hypothesis by the event $H = 0$ corresponding to the case when there is no signal. That is, for all n , X_n are i.i.d., exponentially distributed

$$\mathbb{E}[X_n] = 1 \quad \text{for all } n. \quad (2)$$

The case of null hypothesis ($H = 0$) is identical to (1) for the case where the SNR is $\gamma = 0$. We will denote by $H = 1$ the event that there is a signal in some signal region S with some SNR γ .

Second, if a signal is detected (that is, it is estimated that $H = 1$), we wish to estimate the SNR γ and the signal interval S .

B. Multi-Dimension Extension

The problem is naturally extended to dimensions $d \geq 1$. In this case, $\mathbf{X}$ is a tensor of d -th order with components $X_{n_1, \dots, n_d}$ with $n_i \in [0, N_i]$. For example, $\mathbf{X}$ could be measurements over N_0 time bins and N_1 frequency bins, as you may obtain from N_0 FFTs, each FFT of length N_1 . In the multi-dimensional case, the set S is a hyper-cube:

$$S = \{(n_1, \dots, n_d) \mid a_i \leq n_i < b_i \ \forall i = 1, \dots, d\}, \quad (3)$$

for some left and right intervals $\mathbf{a} = (a_1, \dots, a_d)$ and $\mathbf{b} = (b_1, \dots, b_d)$. As an example, Fig. 1(b) shows a $d = 2$ case for a time-frequency search. The signal region corresponds to a bounding box in time and frequency. Again, we assume that, given S , $X_{n_1, \dots, n_d}$ are exponential and independent with:

$$\mathbb{E}[X_{n_1, \dots, n_d}] = \begin{cases} 1 + \gamma & \text{if } (n_1, \dots, n_d) \in S \\ 1 & \text{if } (n_1, \dots, n_d) \notin S, \end{cases} \quad (4)$$

where, again, γ represents an SNR. To use a uniform notation between the vector and tensor cases, we will let $\mathbf{n} = (n_1, \dots, n_d)$ be a d -dimensional tensor index and write the components of the tensor as $X_{\mathbf{n}} = X_{n_1, \dots, n_d}$.

Again, the problem in the multi-dimensional case is to detect if a signal is present, and if so, to estimate the signal region S and SNR $\gamma > 0$.

III. LIKELIHOOD RATIO TEST

A. Likelihood Function

We follow a standard likelihood ratio detector for the signal detection problem. Let $H = 0, 1$ denote the true signal hypothesis:

$$H = \begin{cases} 0 & \text{no signal present;} \\ 1 & \text{signal is present.} \end{cases} \quad (5)$$

We compute an estimate $\hat{H} = 0, 1$ of H as follows: Let $p(\mathbf{X}|S, \gamma)$ denote the PDF of the measurements of $\mathbf{X}$ for a given signal region S and SNR γ . Also, let $p_0(\mathbf{X}) := p(\mathbf{X}|\emptyset, 0)$ denote the conditional PDF for the null hypothesis $H = 0$ when the signal is not present (i.e., no SNR). Let $J(S, \gamma)$ denote the log likelihood ratio

$$J(S, \gamma) := \log \left[\frac{p(\mathbf{X}|S, \gamma)}{p_0(\mathbf{X})} \right]. \quad (6)$$

Since the parameters γ and S are not known, we use a modified version of the Generalized Likelihood Ratio Test (GLRT) [10], [11]: We compute the maxima:

$$J_\ell^* := \underset{\substack{S, \gamma \geq 0 \\ |S| = \ell}}{\operatorname{argmax}} J(S, \gamma) \quad (7)$$

where $|S|$ is the cardinality (number of elements) of S . Hence, J_ℓ^* is the maxima over all $\gamma > 0$ and sets S with $|S| = \ell$. We then assume a detector of the form:

$$\hat{H} = \begin{cases} 1 & J_\ell^* \geq t_\ell \text{ for some } \ell \\ 0 & J_\ell^* < t_\ell \text{ for all } \ell, \end{cases} \quad (8)$$

where t_ℓ are a set of thresholds that depend on the set size S . In the standard GLRT, the threshold levels t_ℓ are equal.

However, as we will see below, having a set size-dependent threshold will enable a simpler analysis for the false alarm probability.

For each ℓ , we can compute J_ℓ^* in (7), in two steps: First, for each S , we maximize the likelihood ratio $J(S, \gamma)$ over γ :

$$J(S) := \max_{\gamma \geq 0} J(S, \gamma). \quad (9)$$

Then, J_ℓ^* can be computed by maximizing over S :

$$J_\ell^* = \max_{|S| = \ell} J(S). \quad (10)$$

Our first lemma provides a simple expression for the likelihood ratio and maximization over γ :

Lemma 1. *The likelihood is given by:*

$$J(S, \gamma) = |S| \left[\overline{X}_S \left[\frac{\gamma}{1 + \gamma} \right] - \log(1 + \gamma) \right]. \quad (11)$$

where $\overline{X}_S$ is the average value of $X_{\mathbf{n}}$ on the set S :

$$\overline{X}_S = \frac{1}{|S|} \sum_{\mathbf{n} \in S} X_{\mathbf{n}}, \quad (12)$$

The likelihood maximized over the SNR is given by:

$$J(S) = |S| \left[\overline{X}_S^+ - 1 - \log(\overline{X}_S^+) \right]. \quad (13)$$

where

$$\overline{X}_S^+ = \max\{\overline{X}_S, 1\}. \quad (14)$$

Proof. See Appendix A. $\square$

Using this lemma, we can rewrite the GLRT as follows: Let $\varphi(x)$ be the function:

$$\varphi(x) = x - 1 - \log(x), \quad x \geq 1. \quad (15)$$

Lemma 1 shows that the GLRT (8) can be written as $\hat{H} = 1$ when

$$\ell \varphi(\overline{X}_S^+) \geq t_\ell \text{ for some } |S| = \ell. \quad (16)$$

Since $\varphi(x)$ is increasing for $x \geq 1$, we can define

$$u_\ell = \varphi^{-1}(t_\ell/\ell), \quad (17)$$

and the GLRT estimator (8) is equivalent to:

$$\hat{H} = \begin{cases} 1 & X_S^+ \geq u_{|S|} \text{ for some } S \\ 0 & X_S^+ < u_{|S|} \text{ for all } S. \end{cases} \quad (18)$$

B. False Alarm Probability

Our next results bounds the false alarm probability

$$P_{\text{FA}} = \mathbb{P}(\hat{H} = 1 | H = 0) \quad (19)$$

as a function of the threshold levels u_ℓ in the GLRT (18). This bound will allow us to select the threshold levels. To state the bound, consider the modified GLRT detector (8) (or, equivalently, the GLRT detector in (18)) over a set of hyper-cube regions in (3). Define

$$N = N_1 N_2 \cdots N_d. \quad (20)$$

Each candidate signal region S is defined by a hyper-cube (3) with boundaries $\mathbf{a}$ and $\mathbf{b}$. Since there are N in (20) choices

for $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{b}_i \geq \mathbf{a}_i$, so there are at most $\frac{N^2}{2}$ possible set hyper-cube S . We can then apply a union bound over these set of hypotheses to bound the false alarm probability.

Lemma 2. Consider the modified GLRT detector (18) over a set of hyper-cube regions in (3). Assume that all thresholds satisfy $u_i \geq 0$. Then, the probability of false alarm (19) is bounded above by

$$P_{\text{FA}} \leq \frac{N^2}{2} \max_{\ell=1, \dots, N} F(2\ell u_\ell; 2\ell), \quad (21)$$

where N is defined in (20) and $F(s, \nu)$ is the complementary CDF for a chi-squared random variable with ν degrees of freedom. That is,

$$F(s; \nu) = \mathbb{P}(Z_\nu \geq s), \quad (22)$$

where Z_ν is a chi-squared random variable with ν degrees of freedom.

Proof. See Appendix B. $\square$

The lemma 2 provides a simple recipe for computing the threshold levels u_ℓ for the GLRT detector in (18), or equivalently, the threshold levels t_ℓ for the GLRT detector in (8). First, we set a desired false alarm probability target P_{FA} . Then, each u_ℓ should satisfy

$$\frac{2P_{\text{FA}}}{N^2} = F(2\ell u_\ell; 2\ell). \quad (23)$$

We solve (23) by first computing value u_ℓ with the inverse complementary CDF:

$$u_\ell = \frac{1}{2\ell} F^{-1}\left(\frac{2P_{\text{FA}}}{N^2}; 2\ell\right), \quad (24)$$

Then, we compute t_ℓ from (17)

$$t_\ell = \ell \varphi(u_\ell). \quad (25)$$

C. Probability of Missed Detection

The probability of missed detection is

$$P_{\text{MD}} = \mathbb{P}(\hat{H} = 0 | H = 1), \quad (26)$$

which is generally a function of the true signal region S and SNR γ . The following lemma provides a bound on the missed detection probability.

Lemma 3. If the true signal region is S and true SNR is γ , the probability of missed detection is bounded by:

$$P_{\text{MD}} \leq 1 - F\left(\frac{2\ell u_\ell}{1 + \gamma}; 2\ell\right), \quad (27)$$

where $\ell = |S|$, where, as before $F(x; \nu)$ is the complementary CDF (22) of a chi-squared random variable with ν degrees of freedom.

Proof. See Appendix C. $\square$

IV. COMPUTATIONALLY EFFICIENT BINARY SEARCH

A. Dyadic Interval Detection

The GLRT detector in (8) requires searching over all possible signal regions S . We will call this method *exhaustive ML*.

As discussed above, there are $\approx N^2/2$ such signal regions where N is given by the product (20). Hence, the complexity of exhaustive ML will be $O(N^2)$, which may be challenging, particularly at high sample rates.

In this section, we propose a simplified search method for the linear search case where $d = 1$. Exhaustive ML in this case has complexity $O(N^2)$. The proposed method will have complexity $O(N)$.

The key to the algorithm is to perform a binary search on a set of *dyadic intervals*. We assume $N = 2^M$ for some $M \geq 0$. The dyadic intervals are

$$I_{m,i} = [i2^m, (i+1)2^m) \subseteq [0, 1, \dots, N), \quad (28)$$

where $m = 0, \dots, M$ and $i = 0, \dots, 2^{M-m} - 1$. We will call $I_{m,i}$ the i -th dyadic interval at the m -th level. The proposed algorithm, which we call *binary search*, operates in two phases:

- *Initial interval detection* where we search the set of dyadic intervals to detect if a signal is present.
- *Binary interval estimation*: If a signal is detected in the dyadic interval search, the most likely interval is estimated using a binary search.

Algorithm 1 Initial Interval Search

Require: Interval length $N = 2^M$

Require: Power measurements $X_n, n = 0, \dots, N - 1$

Require: Threshold levels $u_k, k = 1, 2, 4, \dots, 2^M$

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1:  $\hat{H} \leftarrow 0$ 
2: for  $m = 0, \dots, M$  do
3:    $\ell \leftarrow 2^m$  // interval length
4:    $k \leftarrow 2^{M-m}$  // number of intervals
5:    $Z_m^* \leftarrow 0$ 
6:   for  $i = 0, \dots, k - 1$  do
7:     if  $m = 0$  then
8:        $Z_{0,i} \leftarrow X_i$ 
9:     else
10:       $Z_{m,i} \leftarrow (Z_{m-1,2i} + Z_{m-1,2i+1})/2$ 
11:    end if
12:    if  $Z_{m,i} \geq u_k$  then
13:       $Z_m^* \leftarrow Z_{m,i}$ 
14:       $i_m^* \leftarrow i$ 
15:    end if
16:  end for
17:  if  $Z_m^* \geq u_k$  then
18:     $\hat{H} \leftarrow 1$ 
19:     $J_m^* \leftarrow \ell(Z_m^* - 1 - \log(Z_m^*))$ 
20:  end if
21: end for
22: if  $\hat{H} = 1$  then
23:    $\hat{m} \leftarrow \operatorname{argmax}_m J_m^*$ 
24:    $\hat{i} \leftarrow i_m^*$ 
25:   return  $(\hat{H} = 1, \hat{m}, \hat{i})$ 
26: else
27:   return  $\hat{H} = 0$ 
28: end if
```

In this subsection, we describe the first stage, namely

the dyadic interval search. The precise steps are shown in Algorithm 1. In this algorithm, we start with N intervals, each representing a power measurement bin X_i . At each stage, we sequentially merge adjacent intervals and compute X_S for all resulting intervals. It can be verified that:

$$Z_{m,i} = \bar{X}_S \text{ for } S = I_{m,i}, \quad (29)$$

so the algorithm computes the mean energy on all the dyadic intervals. The resulting estimate is then:

$$\hat{H} = \begin{cases} 1 & \text{if } \bar{X}_S \geq u_{|S|} \text{ for some } S = I_{m,i} \\ 0 & \text{if } \bar{X}_S < u_{|S|} \text{ for all } S = I_{m,i}. \end{cases} \quad (30)$$

Hence, Algorithm. 1 produces a hypothesis estimate $\hat{H}$ that matches the estimate for the GLRT detector (18), except that the search is only over the dyadic intervals $S = I_{m,i}$ as opposed to all subsets.

B. Interval Estimation

If a signal has been detected by Algorithm 1, we then obtain an estimate for the interval by Algorithm 2. This algorithm takes as inputs the binary energy values $Z_{m,i}$ from Algorithm. 1. If the signal length is $N = 2^M$, the algorithm proceeds in M iterations indexed by $t = 0, \dots, M-1$. In each iteration, it finds an estimate for the interval of the form:

$$\hat{S}^{(t)} = [\hat{a}^{(t)}, \hat{b}^{(t)}], \quad (31)$$

where the left and right boundaries $\hat{a}^{(t)}$ and $\hat{b}^{(t)}$ are of the form:

$$\hat{a}^{(t)} = \hat{i}^{(t)} \ell_t, \quad \hat{b}^{(t)} = \hat{j}^{(t)} \ell_t, \quad \ell_t = 2^{M-t}, \quad (32)$$

so that they are restricted to discrete values:

$$\hat{a}^{(t)}, \hat{b}^{(t)} \in \{0, \ell_t, 2\ell_t, \dots, N\}.$$

We call the spacing ℓ_t the *sub-interval length* in iteration t . Hence, the algorithm finds iteratively finer estimates of the interval boundaries since the sub-interval length ℓ_t decreases as t increases. Initially, the algorithm takes the estimates

$$\hat{S}^{(0)} = [\hat{a}^{(0)}, \hat{b}^{(0)}] = [0, N], \quad (33)$$

corresponding to the entire interval.

At each iteration, it then iteratively refines first the estimate for the left boundary $\hat{a}^{(t)}$ and then the right boundary $\hat{b}^{(t)}$. The update for the left boundary searches over candidates:

$$\hat{a}^{(t+1)} \in \{\hat{a}^{(t)} - \ell_{t+1}, \hat{a}^{(t)}, \hat{a}^{(t)} + \ell_{t+1}\} \quad (34)$$

so that it searches over one index to the right and left. For each candidate, $\hat{a}^{(t+1)}$, the algorithm evaluates the objective $J([\hat{a}^{(t+1)}, \hat{b}^{(t)}])$ where $J(S)$ is the likelihood in (13). Similarly, the optimization over the right boundary searches over values for $\hat{b}^{(t+1)}$ in the set:

$$\hat{b}^{(t+1)} \in \{\hat{b}^{(t)} - \ell_{t+1}, \hat{b}^{(t)}, \hat{b}^{(t)} + \ell_{t+1}\}, \quad (35)$$

so that it searches over one index to the left and right.

Algorithm 2 Binary Interval Estimation

Require: Interval length $N = 2^M$

Require: Dyadic powers $Z_{m,i}$

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1:  $\hat{i}^{(0)} \leftarrow 0$  // initial left index
2:  $\hat{j}^{(0)} \leftarrow 1$  // initial right index
3:  $\hat{Z}^{(0)} = Z_{M,0}$ , // initial average energy
4:  $\hat{L}^{(0)} \leftarrow N = 2^M$  // initial interval length
5:  $\ell_0 \leftarrow 2^M$  // initial sub-interval length
6:  $Z^+ = \max\{\hat{Z}^{(0)}, 1\}$ 
7:  $J_{\max} = \hat{L}^{(0)}[Z^+ - 1 - \ln(Z^+)]$  // initial objective
8:
9: for  $t = 0, \dots, M-1$  do
10:   // Get layer info
11:    $m \leftarrow M - t - 1$  // layer
12:    $\ell_{t+1} \leftarrow 2^m$  // sub-interval length
13:
14:   // Optimization over left boundary
15:    $\tilde{Z}^{(t)} \leftarrow \hat{Z}^{(t)}$ ,  $\hat{i}^{(t+1)} \leftarrow 2\hat{i}^{(t)}$ ,  $\tilde{L}^{(t)} \leftarrow \hat{L}^{(t)}$ 
16:   for  $\delta \in \{-1, 1\}$  do
17:      $i' \leftarrow 2\hat{i}^{(t)} + \delta$ 
18:      $L' \leftarrow \tilde{L}^{(t)} - \delta\ell_{t+1}$ 
19:      $Z' \leftarrow (\tilde{L}^{(t)}\tilde{Z}^{(t)} - \delta\ell_{t+1}Z_{m,i'})/L'$ 
20:      $J' \leftarrow L'[(Z^+ - 1) - \ln(Z^+)]$ ,  $Z^+ = \max\{Z', 1\}$ 
21:     if  $J' > J_{\max}$ ,  $L' \geq 0$ , and  $i' \geq 0$  then
22:        $J_{\max} \leftarrow J'$ 
23:        $\hat{i}^{(t+1)} \leftarrow i'$ 
24:        $\tilde{Z}^{(t)} \leftarrow Z'$ ,  $\tilde{L}^{(t)} \leftarrow L'$ 
25:     end if
26:   end for
27:
28:   // Optimization over right boundary
29:    $\tilde{Z}^{(t+1)} \leftarrow \tilde{Z}^{(t)}$ ,  $\hat{j}^{(t+1)} \leftarrow 2\hat{j}^{(t)}$ ,  $\hat{L}^{(t+1)} \leftarrow \tilde{L}^{(t)}$ 
30:   for  $\delta \in \{-1, 1\}$  do
31:      $j' \leftarrow 2\hat{j}^{(t)} + \delta$ 
32:      $L' \leftarrow \tilde{L}^{(t)} + \delta\ell_{t+1}$ 
33:      $Z' \leftarrow (\tilde{L}^{(t)}\tilde{Z}^{(t)} + \delta\ell_{t+1}Z_{m,j'})/L'$ 
34:      $J' \leftarrow L'[(Z^+ - 1) - \ln(Z^+)]$ ,  $Z^+ = \max\{Z', 1\}$ 
35:     if  $J' > J_{\max}$ ,  $L' > 0$ , and  $j' \leq 2^{t+1}$  then
36:        $J_{\max} \leftarrow J'$ 
37:        $\hat{j}^{(t+1)} \leftarrow j'$ 
38:        $\tilde{Z}^{(t+1)} \leftarrow Z'$ ,  $\hat{L}^{(t+1)} \leftarrow L'$ 
39:     end if
40:   end for
41: end for
```

C. Complexity

The algorithm 1 comprises $\log_2(N)$ steps, within each of which the energy is assessed across $N/2^m$ intervals. Consequently, $O(N)$ computations are required in total to determine the energy of all intervals. For Algorithm 2, there are two key computational savings. First, each evaluation of $J(S)$ does not require recomputing the average energy $\bar{X}_S$. Recomputing $\bar{X}_S$ directly requires taking a sum over $|S|$ elements which can be as large as N . However, the algorithm uses the pre-computed values $Z_{m,i}$ to compute $\bar{X}_S$. Secondly, the search involves only $M = \log_2(N)$ iteration and, in each iteration,

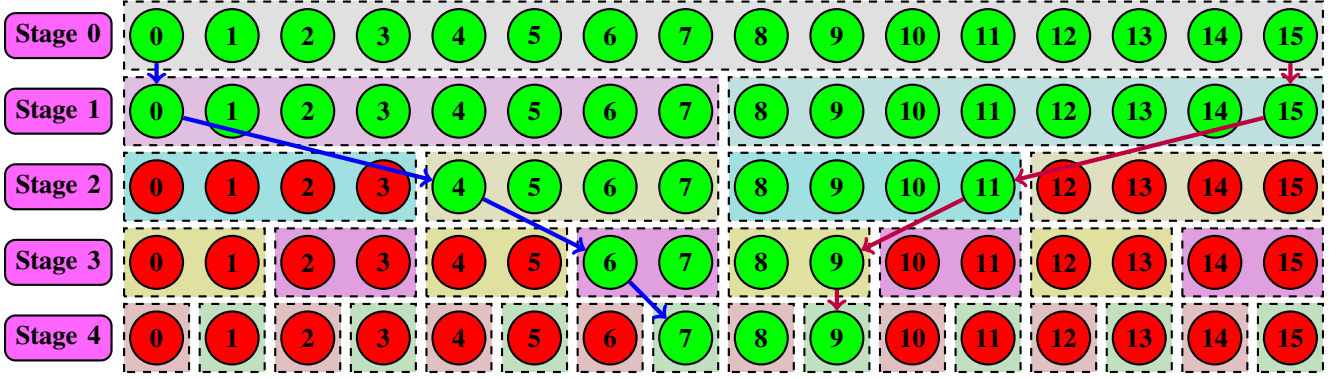


Fig. 2: The diagram illustrates the computationally efficient maximum likelihood method using binary search over possible intervals. The figure demonstrates the method for $N = 16$, where N represents the number of power measurements. The blue and purple lines respectively depict the trajectory of the detected start and end of the optimal interval across different stages. The signal, detected within the interval $X_{[7,9]}$, is highlighted by green circles.

there are a bounded number of operations. Hence, the overall complexity of Algorithm 2 is $O(\log_2(N))$. So, the proposed method, which is the union of the two algorithms, has an overall complexity of $O(N)$.

V. ASYMPTOTIC CONSISTENCY OF THE BINARY SEARCH

In this section, we prove that the binary interval estimation algorithm, Algorithm 2, is asymptotically consistent in the limit of large N under a fixed SNR per bin γ . Specifically, we consider the $d = 1$ dimensional case and consider a sequence of problems indexed by the total length N . We fix a “true” SNR γ and assume that the true signal interval

$$S_N^0 = [a_N^0, b_N^0], \quad a_N^0 = \lfloor \alpha_0 N \rfloor, \quad b_N^0 = \lfloor \beta_0 N \rfloor, \quad (36)$$

for some constants $0 < \alpha_0 < \beta_0 < 1$ that do not depend on N . Assume that X_n , $n = 0, \dots, N$ are generated i.i.d. from the model (1). We let $\hat{a}_N^{(t)}, \hat{b}_N^{(t)}$ be the outputs of Algorithm. 2, and let $\hat{S}_N^{(t)}$ denote the corresponding interval:

$$\hat{S}_N^{(t)} = [\hat{a}_N^{(t)}, \hat{b}_N^{(t)}] \quad (37)$$

A common metric for the performance of the algorithm is the so-called intersection over union or IoU:

$$\text{IoU}_N^{(t)} = \frac{|\hat{S}_N^{(t)} \cap S_N^0|}{|\hat{S}_N^{(t)} \cup S_N^0|}. \quad (38)$$

Although the true interval is modeled as deterministic, the estimated interval $\hat{S}_N^{(t)}$ and, hence, the performance metric $\text{IoU}_N^{(t)}$ is random.

Theorem 4. *Under the above assumptions*

$$\lim_{N, t \rightarrow \infty} \text{IoU}_N^{(t)} = 1 \quad (39)$$

almost surely.

Proof. See Appendix D $\square$

This asymptotic consistency result (Theorem 4) is important because it establishes that the proposed binary interval estimation algorithm reliably identifies the correct signal interval in the limit of large measurement sizes. Practically, this means

that as more data is collected (larger N) and the algorithm is allowed sufficient refinement steps (larger t), the estimated interval converges exactly to the signal region identified by the exhaustive algorithm, guaranteeing high accuracy in signal detection tasks.

VI. BASELINE NEURAL NETWORKS

It is important to compare the proposed algorithm to widely-used U-Net networks. The U-Net architecture is widely used in segmentation and detection tasks [47]–[49], and has also been proposed for spectral segmentation and signal detection [50]–[54]. Our simulations below will show that the proposed method offers both improved complexity and performance over U-Nets.

For the one-dimensional case, we consider the U-Net depicted in Fig. 3. For higher-dimensional data, the standard U-Net architecture is employed, with a sufficient number of encoding and decoding stages and appropriately scaled data and filter sizes. In each stage, a two-step convolutional block is applied. This block consists of a convolution operation, followed by Batch Normalization and ReLU activation, and is concluded with a MaxPooling layer to downsample the signal by a factor of two. During the encoding stages, feature map information is progressively captured across multiple channels, culminating in M channels of 1×1 data. This process parallels the proposed binary search method. Initially, $J(S)$ is computed over the entire interval, similar to the first stage of the U-Net encoder. The search is then refined over progressively smaller intervals, analogous to the subsequent encoder stages of the U-Net, until the signal is examined in each bin, corresponding to the final stage of the U-Net encoder.

In the decoding phase, similar to the conventional U-Net architecture, we iteratively up-sample the feature maps and concatenate them with the corresponding encoder feature maps. A similar convolutional block is then applied to the concatenated feature maps. This process continues until we obtain a two-channel output segmentation map, where a $\text{Conv}1 \times 1$ layer is used to generate the final segmentation mask. Intuitively, the decoder seeks to reconstruct the signal interval information

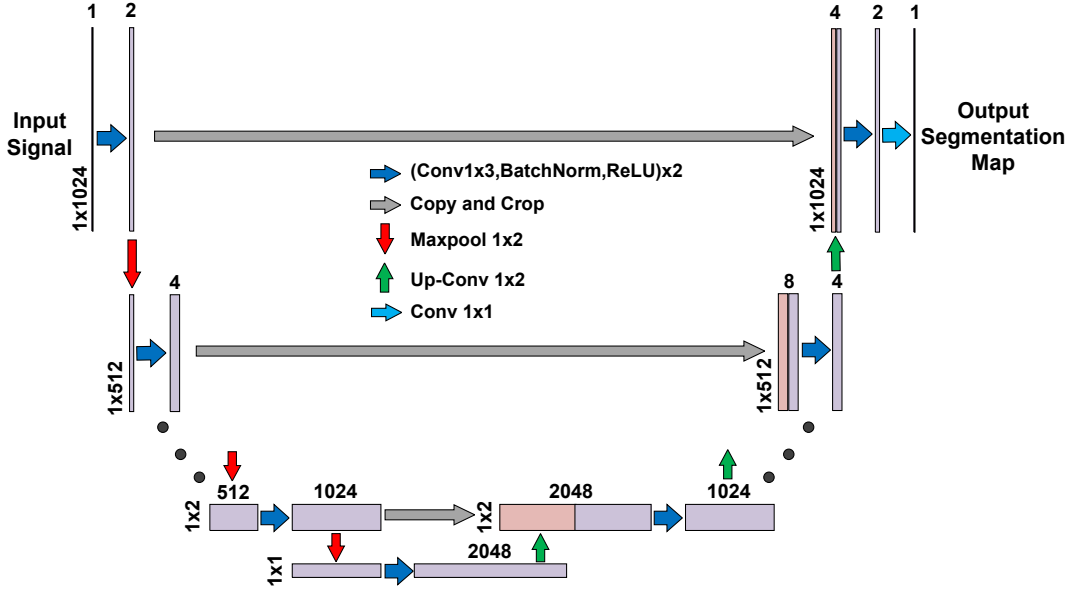


Fig. 3: The architecture of the baseline U-Net used for signal detection in this paper. This architecture is designed for one-dimensional data. However, the same principles and methods can be extended to handle higher-dimensional data. For each feature map, dimensions are annotated at the bottom left, while the number of channels is indicated at the top. As illustrated in the figure, the process initiates with the input signal, upon which a convolutional block comprising two stages of 1×3 convolution, Batch Normalization, and ReLU activation is applied. This is followed by a Max Pooling operation, which is iteratively repeated until achieving a 1×1 feature map with 2048 channels. The decoding phase commences with up-convolutions, wherein feature maps are concatenated with the corresponding encoder maps, applying the same convolutional block utilized during the encoding stage. Finally, a $\text{Conv} 1 \times 1$ layer is employed to generate the output segmentation map.

from the encoded channels using convolutional blocks, which parallels the functionality of our binary search algorithm.

VII. EXPERIMENTAL RESULTS

Our goal is to compare the detection accuracy and signal set estimation error of the proposed binary search method to more complex exhaustive ML. Additionally, for the signals that are detected, we will compare the estimation error to a trained U-Net. We consider two scenarios for these comparisons:

- *1D detection*: We take $N = 1024$ points. When a signal is present, it is in an interval with variable length ℓ and variable SNR.
- *2D detection*: We take $N = 128 \times 128$ points. When a signal is present, it is on a square of size $\ell \times \ell$ and variable SNR.

We train U-Nets for each of the two cases. In both cases, we generate $n = 200,000$ samples. Since the U-Net is only used for the estimation of the signal set, all n samples have a signal present. Hence, each sample i can be represented as a pair (X_i, S_i) where X_i is the vector or array of power values and S_i is the true signal interval. We vary the SNR in the samples from -3 dB and 20 dB, with an emphasis on lower SNRs. For the 1D case, the interval size is varied from 1 to 256 and in the 2D case ℓ is varied from 1 to 64. The dataset is divided into training and test sets with an 80/20% ratio. A summary of the hyperparameters and settings used for training and evaluation of the U-Net is provided in Table. I.

TABLE I: Summary of hyperparameters and settings used for U-Net training and evaluation.

Hyperparameter	Value
Optimization Algorithm	Adam
Initial Learning Rate	10^{-2}
Learning Rate Scheduler	StepLR
Batch Size	64
Number of Training Epochs	50
Training Loss Function	Binary Cross-Entropy (BCE)
Evaluation Metric	Intersection over Union (IoU)
U-Net Parameters (1D)	43,380,007
U-Net Parameters (2D)	1,949,011
PyTorch Version	2.6.0

For detection accuracy, we fix the threshold for a false alarm rate of 10^{-6} using the bound in Lemma 2. Figures. 5 and 7 show the missed detection for exhaustive ML and binary search methods. We see that in the 1D case, the detection is no more than 3 dB worse and no more than 6 dB worse in the 2-d case. This difference is explicable since, in the worst case, the binary search captures $1/2$ the energy in the 1D case and $1/4$ the energy in the 2D case. Importantly, as the interval sizes increase, the gap decreases, indicating that the proposed method has minimal loss for larger interval sizes.

Figures. 4 and 6 show the estimation accuracy for the detected signals. Here, we compare the exhaustive ML, binary search, and U-Net. For all three, we plot the detection

TABLE II: Comparison of number of floating point operations (FLOPs) needed for the proposed binary search method, exhaustive ML method, and U-Net

Algorithm	Number of FLOPs	
	1D-1024	2D-128×128
Exhaustive ML	5.24×10^6	7.27×10^8
U-Net	4.0×10^7	10^7
Binary Search (Proposed Method)		
Algorithm 1	3.13×10^3	2.86×10^4
Algorithm 2	6.2×10^2	9.1×10^2
Binary Search-Total	3.75×10^3	2.95×10^4

intersection over union (IOU) error rate, defined as:

$$\text{IOU Error Rate} = 1 - \frac{S \cap \hat{S}}{S \cup \hat{S}} \quad (40)$$

where S represents the ground-truth interval and $\hat{S}$ corresponds to the estimated interval. From the figures, we see that for any SNR, as the interval sizes increase, the detection error rate goes to zero, as predicted by Theorem 4. Second, for larger interval sizes the gap between the exhaustive ML and binary search decreases, again suggesting that the proposed method will work well for larger intervals.

Finally, the proposed method significantly outperforms the U-Net in most interval sizes. This improvement in performance occurs even though the proposed binary search method is significantly simpler. As a way to compare, we can count the number of operations for detection (Algorithm. 1), binary interval estimation (Algorithm. 2), exhaustive ML and U-Net. Table. II presents a comprehensive summary of the floating point operations required for each algorithm considered. These figures were estimated using a Python-based simulator. As evidenced by the data in the table, the proposed binary search algorithm requires significantly fewer operations -by several orders of magnitude - compared to both U-Net and exhaustive ML methods, applicable to both 1D and 2D scenarios.

VIII. CONCLUSION

In this paper, we investigate signal detection challenges in environments characterized by unknown signal bandwidth and occupancy intervals, particularly under adversarial and spectrum-sharing conditions. We introduced an effective Generalized Likelihood Ratio Test (GLRT)-based approach that leverages normalized average signal energy as a straightforward but powerful detection metric. Our theoretical analysis provided bounds for false alarm and missed detection probabilities, explicitly linking them to the signal-to-noise ratio (SNR) and the size of the signal set. Addressing the significant computational overhead inherent to exhaustive search methods, we proposed a computationally efficient binary search algorithm that reduces the complexity from $O(N^2)$ to $O(N)$ in one-dimensional scenarios. Notably, this binary search method achieves performance comparable to exhaustive searches and demonstrates asymptotic consistency, ensuring that the interval-of-overlap converges to unity under constant SNR conditions as the measurement size grows. Our comprehensive simulation studies validated the efficacy and efficiency

of the proposed binary search method. The method not only maintained near-optimal performance relative to exhaustive searches, but also demonstrated superior detection capabilities and significantly lower computational complexity compared to contemporary neural network-based approaches, notably outperforming specialized U-Net models. In conclusion, our proposed GLRT-based binary search approach provides a robust and computationally efficient solution for signal detection in uncertain spectral environments, promising substantial practical benefits for spectrum sensing applications, especially in resource-constrained systems. Future work will consider the important case of multiple signals and more realistic models for signals that are not exactly aligned to the degrees of freedom.

APPENDIX A PROOF OF LEMMA 1

Given any signal region S and SNR γ , each X_n is exponentially distributed as

$$p(X_n|S, \gamma) = \lambda_n^{-1} e^{-X_n/\lambda_n}, \quad (41)$$

where

$$\lambda_n = \begin{cases} 1 & \text{if } n \notin S, \\ 1 + \gamma & \text{if } n \in S, \end{cases} \quad (42)$$

Therefore,

$$\begin{aligned} \log \frac{p(\mathbf{X}|S, \gamma)}{p_0(\mathbf{X})} &= \log \prod_{n \in S} \left[\frac{1}{1 + \gamma} \exp(X_n(1 - 1/(1 + \gamma))) \right] \\ &= -|S| \log(1 + \gamma) + \left[\frac{\gamma}{1 + \gamma} \right] \sum_n X_n \\ &= |S| \left[\bar{X}_S \left[\frac{\gamma}{1 + \gamma} \right] - \log(1 + \gamma) \right], \end{aligned}$$

which proves (11). To find the maxima over γ , we take the derivative with respect to γ :

$$\begin{aligned} \frac{1}{1 + \gamma} &= \bar{X}_S \frac{1}{(1 + \gamma)^2} \\ \Rightarrow 1 + \gamma &= \bar{X}_S. \end{aligned} \quad (43)$$

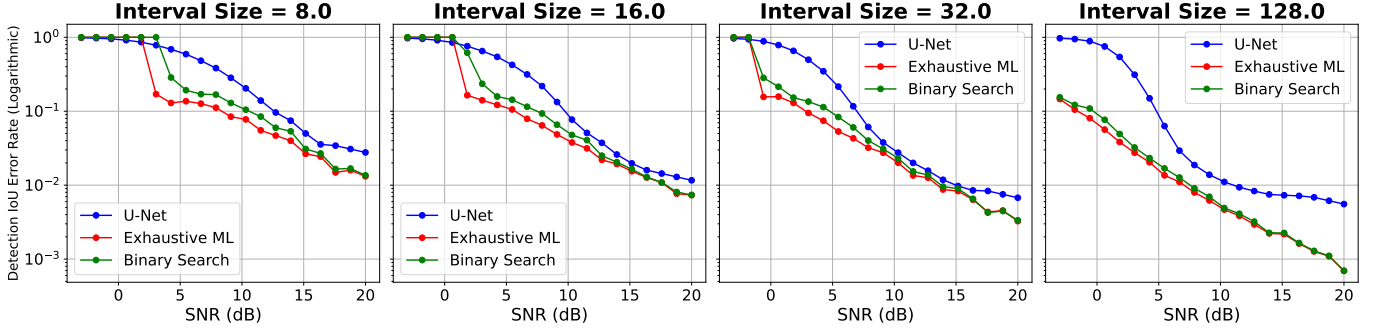
This equation will have a solution $\gamma \geq 0$ if and only if $\bar{X}_S \geq 1$. When $\bar{X}_S \geq 1$, the maximizing value is $\gamma = \bar{X}_S - 1$. Substituting this expression into (11), we obtain:

$$\begin{aligned} J(S) &= \max_{\gamma \geq 0} J(S, \gamma) \\ &= |S| [\bar{X}_S - 1 - \log(\bar{X}_S)] \text{ when } \bar{X}_S \geq 1. \end{aligned} \quad (44)$$

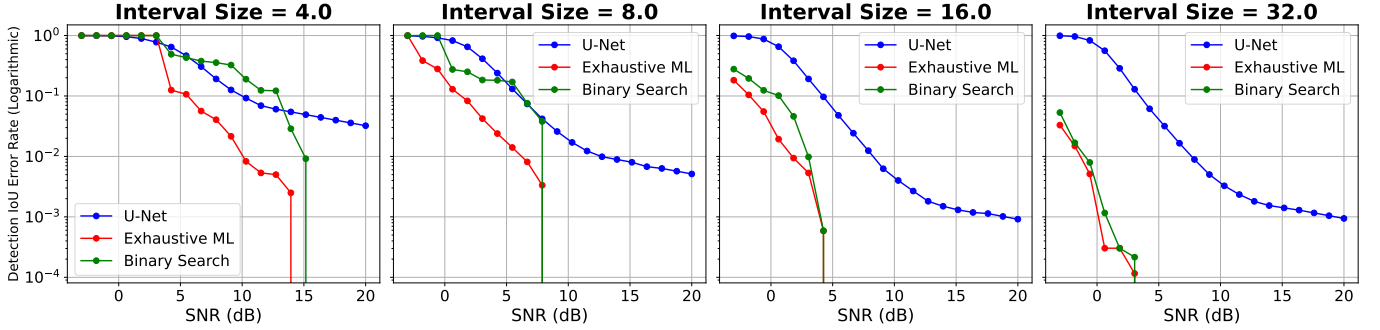
When $\bar{X}_S \leq 1$, the maximum value of $J(S, \gamma)$ occurs when $\gamma = 0$ in which case:

$$J(S) = 0 \text{ when } \bar{X}_S < 1. \quad (45)$$

The formula (13) matches (44) when $\bar{X}_S \geq 1$ (45) when $\bar{X}_S < 1$.



(a) The detection Intersection over Union (IoU) error rate of different methods for different SNRs (false alarm rate fixed at 10^{-6}) for the one-dimensional data.



(b) The detection Intersection over Union (IoU) error rate of different methods for different SNRs (false alarm rate fixed at 10^{-6}) for the two-dimensional data.

Fig. 4: The detection Intersection over Union (IoU) error rate of the exhaustive and binary search maximum likelihood estimations and U-Net for 1D and 2D signals with SNRs ranging from -5 to 20 dB. Each plot represents the performance for a specific fixed signal size. As expected, increasing the SNR improves the performance of both methods. The figures indicate that the performance of the two methods is nearly identical for larger signal sizes. For smaller signal sizes, the binary search method still performs similarly to the exhaustive ML at low and high SNRs, with only a slight, non-significant performance drop observed at intermediate SNRs. In most cases, the binary search ML method surpasses the U-Net in the IoU metric while maintaining significantly lower computational complexity. The performance is generally better for 2D data compared to 1D data.

APPENDIX B PROOF OF LEMMA 2

The proof is a simple application of a union bound:

$$\begin{aligned} P_{\text{FA}} &= \mathbb{P}(\hat{H} = 1 | H = 0) \\ &= \mathbb{P}(\bar{X}_S^+ \geq u_{|S|} \text{ for some } S | H = 0) \\ &\leq \sum_S \mathbb{P}(\bar{X}_S^+ \geq u_{|S|} | H = 0) \end{aligned} \quad (46)$$

Hence, we can approximate the bound (46) as:

$$P_{\text{FA}} \leq \frac{N^2}{2} \max_S \mathbb{P}(\bar{X}_S^+ \geq u_{|S|} | H = 0). \quad (47)$$

Also, from (12), we have that

$$|S| \bar{X}_S = \sum_{n \in S} X_n.$$

Under the null hypothesis, $H = 0$, the values X_n are i.i.d., exponential random variables with mean $\mathbb{E}(X_n) = 1$. Thus,

$$Z_S := 2|S| \bar{X}_S$$

is a chi-squared random variable with $2|S|$ degrees of the freedom [55]. Therefore, for any set S with $|S| = \ell$,

$$\begin{aligned} \mathbb{P}(\bar{X}_S^+ \geq u_{|S|} | H = 0) &= \mathbb{P}(\bar{X}_S^+ \geq u_\ell | H = 0) \\ &= \mathbb{P}(\bar{X}_S \geq u_\ell | H = 0) \\ &= \mathbb{P}(Z_S \geq 2\ell u_\ell | H = 0) = F(2\ell u_\ell; 2\ell). \end{aligned} \quad (48)$$

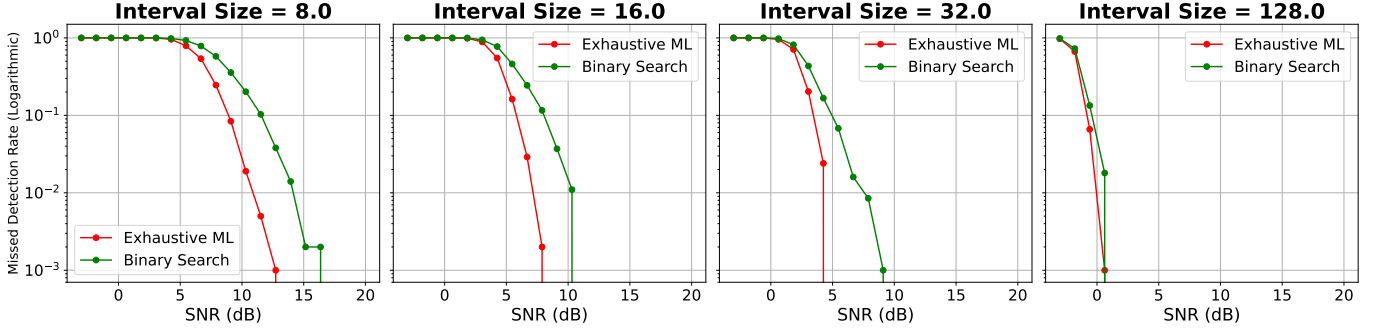
Substituting (48) into (47) we obtain (21).

APPENDIX C PROOF OF LEMMA 3

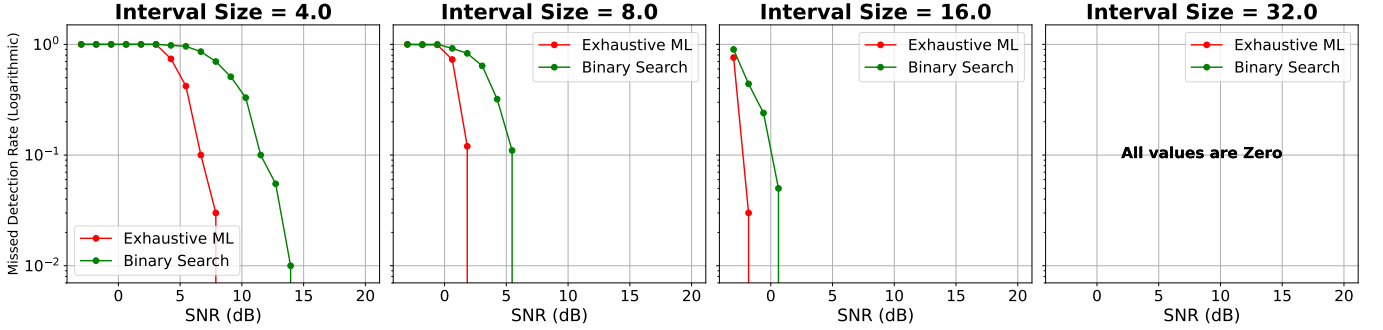
Let S and γ be the true signal region and SNR under the signal present hypothesis $H = 1$. The probability of missed detection is:

$$\begin{aligned} P_{\text{MD}} &= \mathbb{P}(\hat{H} = 0 | H = 1) \\ &= \mathbb{P}(\bar{X}_{S'} < u_{|S'|} \text{ for all } S') \\ &\leq \mathbb{P}(\bar{X}_S \leq u_{|S|}). \end{aligned} \quad (49)$$

That is, we can bound the missed detection probability by the behavior on the true set. Now, under the hypothesis $H = 1$,



(a) The missed detection rate of maximum likelihood methods for different SNRs (false alarm rate fixed at 10^{-6}) for the one-dimensional data.



(b) The missed detection rate of maximum likelihood methods for different SNRs (false alarm rate fixed at 10^{-6}) for the two-dimensional data.

Fig. 5: The missed detection rate of the exhaustive and binary search maximum likelihood estimations and U-Net for 1D and 2D signals with SNRs ranging from -5 to 20 dB. Each plot represents the performance for a specific fixed signal size. As expected, increasing the SNR improves the performance of both methods.

for all elements $n \in S$, X_n is exponentially distributed with $\mathbb{E}(X_n) = (1 + \gamma)$. Hence,

$$Z_S = 2|S|\bar{X}_S/(1 + \gamma)$$

is chi-squared distributed with $2|S|$ degrees of freedom. Therefore, using the complementary CDF in (22), we can write the probability bound in (49) as

$$P_{MD} \leq 1 - F\left(\frac{2\ell u_\ell}{1 + \gamma}; 2\ell\right), \quad (50)$$

where $\ell = |S|$.

APPENDIX D PROOF OF THEOREM 4

A. Equivalent Algorithm

Before proving the theorem, we first rewrite Algorithm. 2 in the form of Algorithm. 3 which is easier to analyze. The following lemma shows these two algorithms are equivalent, and hence we can focus in Algorithm. 3 for the subsequent analysis.

Lemma 5. Fix N and consider any power measurements X_n . Let

$$(\hat{a}_0^{(t)}, \hat{b}_0^{(t)}) = (\hat{a}^{(t)}, \hat{b}^{(t)}) \quad (51)$$

denote the estimates generated from Algorithm. 2, and let

$$(\hat{a}_1^{(t)}, \hat{b}_1^{(t)}) = (\hat{a}^{(t)}, \hat{b}^{(t)}) \quad (52)$$

be the outputs of Algorithm. 3 with input $Z_{m,i}$ in (29). Then, for all $t = 0, \dots, M - 1$:

$$(\hat{a}_1^{(t)}, \hat{b}_1^{(t)}) = (\hat{a}_0^{(t)}, \hat{b}_0^{(t)}). \quad (53)$$

Proof. We use induction for the proof. For $t = 0$, we have:

$$\begin{aligned} \hat{a}_0^{(0)} &= \hat{i}^{(0)}\ell_0 = 0, & \hat{b}_0^{(0)} &= \hat{j}^{(0)}\ell_0 = N, \\ \hat{a}_1^{(0)} &= 0, & \hat{b}_1^{(0)} &= N \end{aligned} \quad (54)$$

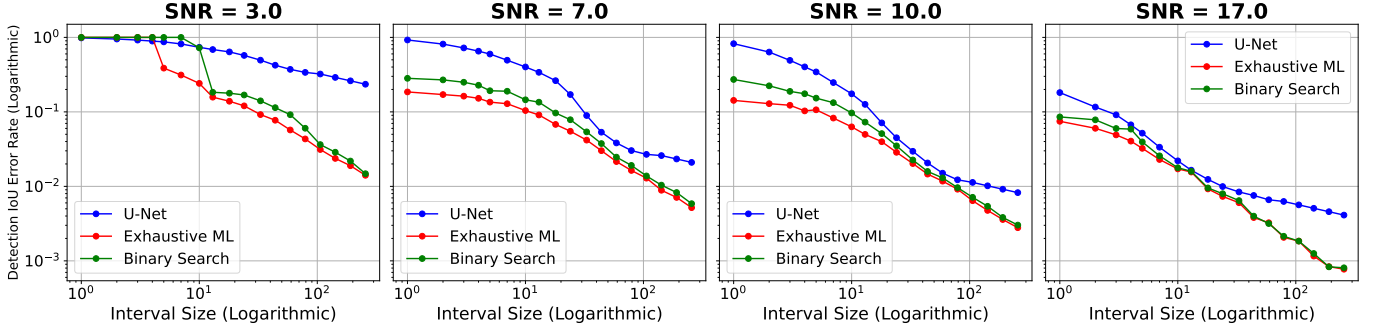
Suppose that the argument holds for step t , so we have:

$$(\hat{a}_1^{(t)}, \hat{b}_1^{(t)}) = (\hat{a}_0^{(t)}, \hat{b}_0^{(t)}). \quad (55)$$

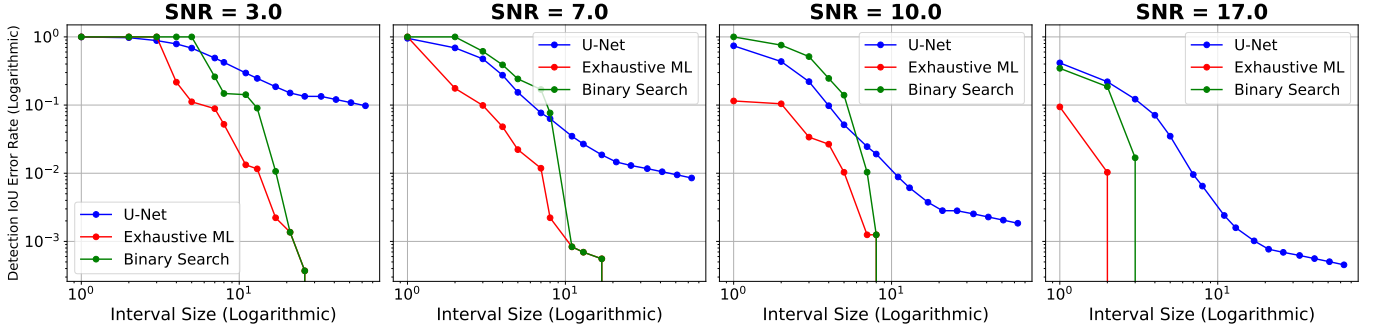
we intend to prove that it will also hold for step $t + 1$. We prove the equivalency of $\hat{a}_0^{(t+1)}$ and $\hat{a}_1^{(t+1)}$, for $\hat{b}$ it will be exactly the same.

$$\begin{aligned} \hat{a}_0^{(t+1)} &= \hat{i}^{(t+1)}\ell_{(t+1)} = (2\hat{i}^{(t)} + \delta_{max})\ell_{(t+1)} \\ \hat{a}_1^{(t+1)} &= \hat{a}_1^{(t)} + \delta_{max}\ell_{(t+1)} \end{aligned} \quad (56)$$

where δ_{max} is the δ which maximizes the objective function J' which is exactly the same for both algorithms. But by assumption, we know that $\hat{a}_1^{(t)} = \hat{a}_0^{(t)}$ and by Algorithm 2 we also know that $\hat{a}_0^{(t)} = \hat{i}^{(t)}\ell_t$ and $\ell_{(t+1)} = 2\ell_t$. So we'll



(a) The detection Intersection over Union (IoU) error rate of different methods for different signal sizes (false alarm rate fixed at 10^{-6}) for the one-dimensional data.



(b) The detection Intersection over Union (IoU) error rate of different methods for different signal sizes (false alarm rate fixed at 10^{-6}) for the two-dimensional data.

Fig. 6: The detection Intersection over Union (IoU) error rate of the exhaustive and binary search maximum likelihood estimations and U-Net for 1D and 2D signals with sizes ranging from 1 to 256 (out of $N = 1024$) for 1D and 1 to 64 (out of $N = 128$) for 2D. Each plot represents the performance for a specific fixed signal SNR. Increasing the signal size improves the performance of both methods, as detecting smaller signals is more challenging. As shown in the figure, the binary search method exhibits a slight, non-significant performance drop, which is less pronounced for larger SNRs. Similar to Fig. 4, the performance of both methods is much closer for very small and very large signal sizes. Similarly in most cases, the binary search ML method outperforms the U-Net in the IoU metric while keeping lower computational complexity. The performance is generally better for 2D data compared to 1D data.

have:

$$\begin{aligned}\hat{a}_1^{(t+1)} &= \hat{a}_1^{(t)} + \delta_{max} \ell_{(t+1)} = \hat{i}^{(t)} 2\ell_{(t+1)} + \delta_{max} \ell_{(t+1)} \\ &= (2\hat{i}^{(t)} + \delta_{max}) \ell_{(t+1)} = \hat{a}_0^{(t+1)}\end{aligned}\quad (57)$$

which proves the equivalency of $\hat{\alpha}_0$ and $\hat{\alpha}_1$. The equivalency of $\hat{\beta}_0$ and $\hat{\beta}_1$ could be proved exactly with the same logic. $\square$

B. Asymptotic Likelihood Function

In this subsection, we next derive a simple expression for the likelihood function in the limit of large N . The likelihood function (13) implicitly depends on N . To make this dependence explicit, we will write $J_N(S)$ for $J(S)$ where the set $S \subseteq \{0, 1, \dots, N-1\}$. Next, for any α, β with

$$0 \leq \alpha \leq \beta \leq 1, \quad (58)$$

we define the sequences

$$a_N = \lfloor \alpha N \rfloor, \quad b_N = \lfloor \beta N \rfloor, \quad (59)$$

and let S_N denote the interval

$$S_N = [a_N, b_N]. \quad (60)$$

Note that, by the definition of (36), S_N^0 is S_N for $\alpha = \alpha^0$ and $\beta = \beta^0$. Also, define the function:

$$G_N(\alpha, \beta) := \frac{1}{N} J_N(\lfloor \alpha N \rfloor, \lfloor \beta N \rfloor), \quad (61)$$

which represents a normalized version of the likelihood function on the interval S_N .

Lemma 6. For any α and β satisfying (58) and

$$\alpha \leq \beta^0 \text{ and } \beta \geq \alpha^0, \quad (62)$$

we have that:

$$\lim_{N \rightarrow \infty} \bar{X}_{S_N} = Z(\alpha, \beta), \quad (63)$$

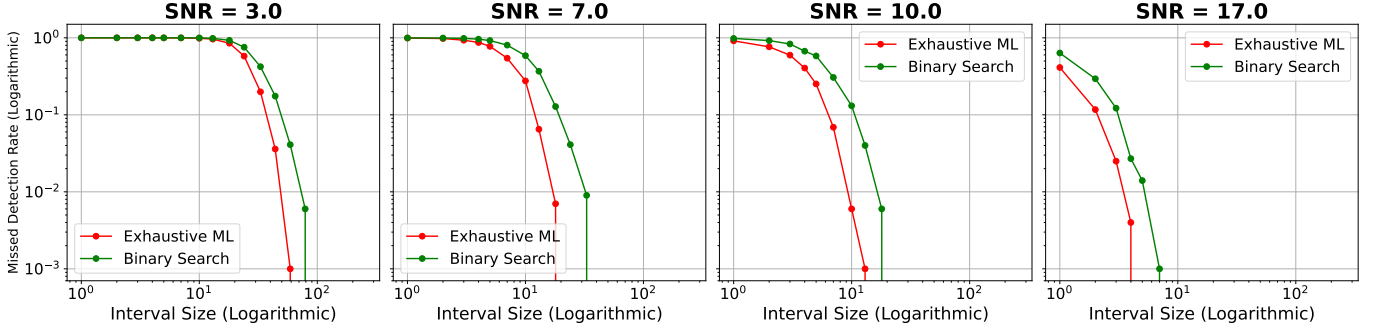
$$\lim_{N \rightarrow \infty} G_N(\alpha, \beta) = G(\alpha, \beta) \quad (64)$$

where the convergence is almost surely and

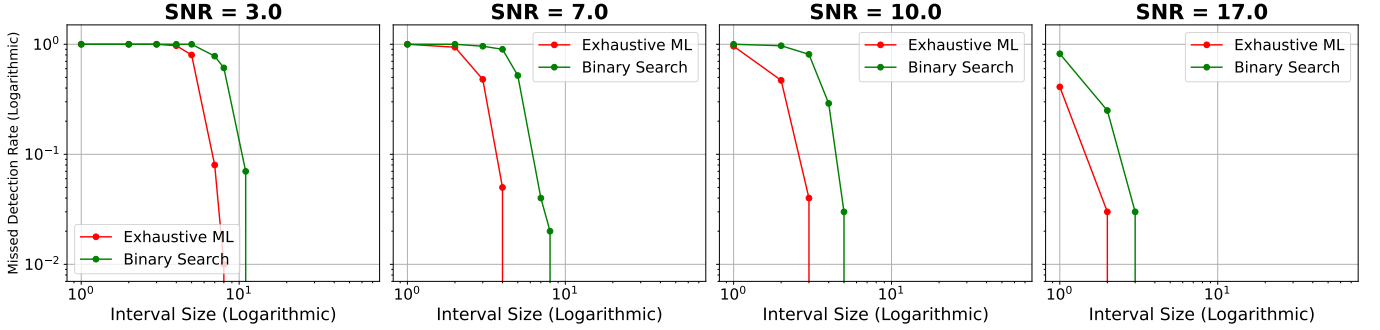
$$Z(\alpha, \beta) := 1 + \gamma \frac{\min\{\beta_0, \beta\} - \max\{\alpha_0, \alpha\}}{\beta - \alpha} \quad (65)$$

$$G(\alpha, \beta) := (\beta - \alpha) [Z(\alpha, \beta) - 1 - \log Z(\alpha, \beta)] \quad (66)$$

Proof. For any S , $\bar{X}_S$ in (12) is the average of values X_n



(a) The missed detection rate of maximum likelihood methods for different signal sizes (false alarm rate fixed at 10^{-6}) for the one-dimensional data.



(b) The missed detection rate of maximum likelihood methods for different signal sizes (false alarm rate fixed at 10^{-6}) for the two-dimensional data.

Fig. 7: The missed detection rate of the exhaustive and binary search maximum likelihood estimations and U-Net for 1D and 2D signals with sizes ranging from 1 to 256 (out of $N = 1024$) for 1D and 1 to 64 (out of $N = 128$) for 2D. Each plot represents the performance for a specific fixed signal SNR.

with $n \in S$. Therefore, the expected values in (1) shows that

$$\mathbb{E}(\bar{X}_S) = 1 + \frac{|S \cap S_N^0|}{|S|} \gamma. \quad (67)$$

From the definition of S_N^0 in (36) and S_N in (59) and (60) as well as the condition (62), we have that

$$\lim_{N \rightarrow \infty} \frac{|S_N \cap S_N^0|}{N} = \min\{\beta_0, \beta\} - \max\{\alpha_0, \alpha\} \quad (68a)$$

$$\lim_{N \rightarrow \infty} \frac{|S_N|}{N} = \beta - \alpha. \quad (68b)$$

Also since $\bar{X}_{S_N}$ is an average of i.i.d. random variables it follows from the strong law of large numbers that

$$\begin{aligned} \lim_{N \rightarrow \infty} \bar{X}_{S_N} &= \lim_{N \rightarrow \infty} \mathbb{E}(\bar{X}_{S_N}) \\ &= 1 + \gamma \lim_{N \rightarrow \infty} \frac{|S \cap S_N^0|}{|S|} \\ &= 1 + \gamma \frac{\min\{\beta_0, \beta\} - \max\{\alpha_0, \alpha\}}{\beta - \alpha} \\ &= Z(\alpha, \beta), \end{aligned} \quad (69)$$

which proves (63). From (61) and (13) we have that:

$$\begin{aligned} G_N(\alpha, \beta) &= \frac{1}{N} J_N([\alpha N], [\beta N]) \\ &= \frac{[\beta N] - [\alpha N]}{N} [\bar{X}_{S_N} - 1 - \log(\bar{X}_{S_N})] \end{aligned} \quad (70)$$

From (63), this limit is given by:

$$\begin{aligned} \lim_{N \rightarrow \infty} G_N(\alpha, \beta) &= (\beta - \alpha) [Z(\alpha, \beta) - 1 - \log Z(\alpha, \beta)] \\ &= G(\alpha, \beta) \end{aligned} \quad (71)$$

which proves (64). $\square$

We will call the function $G(\alpha, \beta)$ the *asymptotic normalized likelihood* and $Z(\alpha, \beta)$ the *asymptotic average signal energy*. An important property of the asymptotic normalized likelihood function is that it possesses a certain *triangular maximization* property, as given by the following definition.

Definition 7. A scalar-valued function $f(x)$ of a scalar x in some interval A has a *triangular maxima* at $x = x^*$ with constant $c > 0$ if:

- (a) $f'(x) > c$ in the region $x < x^*$ and $x \in A$.
- (b) $f'(x) < -c$ in the region $x > x^*$ and $x \in A$

Note that if x^* is a triangular maxima in an interval A , then $x^* = \operatorname{argmax} f(x)$ for $x \in A$.

Lemma 8. Fix any $\alpha_0 < \beta_0$ and γ , and consider the asymptotic normalized likelihood function $G(\alpha, \beta)$ in (66). There exists a constant c , possibly dependent on γ , such that:

- (a) For fixed $\alpha \leq \beta^0$, $G(\alpha, \beta)$ has a triangular maximum at $\beta = \beta^0$ with constant c in the region $\beta \geq \alpha^0$.
- (b) For fixed $\beta \geq \alpha^0$, $G(\alpha, \beta)$ has a triangular maximum at $\alpha = \alpha^0$ with constant c in the region $\alpha \leq \beta^0$.

Algorithm 3 Equivalent Binary Interval Estimation

Require: Interval length $N = 2^M$
Require: Dyadic powers $Z_{m,i}$

```

1:  $\hat{a}^{(0)} \leftarrow 0, \hat{b}^{(0)} \leftarrow N$ 
2:  $J_{\max} = J([\hat{a}^{(0)}, \hat{b}^{(0)}])$ 
3:  $\ell_0 \leftarrow 2^M$  // initial sub-interval length
4: for  $t = 0, \dots, M - 1$  do
5:   // Get layer info
6:    $m \leftarrow M - t - 1$  // layer
7:    $\ell_{t+1} \leftarrow 2^m$  // sub-interval length
8:
9:   // Optimization over the left boundary
10:   $\hat{a}^{(t+1)} \leftarrow \hat{a}^{(t)}$ 
11:  for  $\delta \in \{-1, 1\}$  do
12:     $a' \leftarrow \hat{a}^{(t)} + \delta \ell_{t+1}$ 
13:     $S' = [a', \hat{b}^{(t)})$ 
14:     $J' \leftarrow J(S')$ 
15:    if  $J' > J_{\max}, |S'| > 0$ , and  $a' \geq 0$  then
16:       $J_{\max} \leftarrow J'$ 
17:       $\hat{a}^{(t+1)} \leftarrow a'$ 
18:    end if
19:  end for
20:
21:  // Optimization over the right boundary
22:   $\hat{b}^{(t+1)} \leftarrow \hat{b}^{(t)}$ 
23:  for  $\delta \in \{-1, 1\}$  do
24:     $b' \leftarrow \hat{b}^{(t)} + \delta \ell_{t+1}$ 
25:     $S' = [\hat{a}^{(t+1)}, b']$ 
26:     $J' \leftarrow J(S')$ 
27:    if  $J' > J_{\max}, |S'| > 0$ , and  $b' \leq N$  then
28:       $J_{\max} \leftarrow J'$ 
29:       $\hat{b}^{(t+1)} \leftarrow b'$ 
30:    end if
31:  end for
32: end for
```

Proof. We prove the first part, with a fixed α ; the second part can be proved similarly. We can write $Z(\alpha, \beta)$ as:

$$Z(\alpha, \beta) = \begin{cases} 1 + \gamma \frac{\beta_0 - \hat{\alpha}}{\beta - \alpha}, & \text{if } \beta \geq \beta_0 \\ 1 + \gamma \frac{\hat{\alpha} - \alpha}{\beta - \alpha}, & \text{if } \beta < \beta_0 \end{cases} \quad (72)$$

where $\hat{\alpha} = \max\{\alpha, \alpha^0\}$. So the derivatives of Z with respect to β will be:

$$\frac{\partial Z}{\partial \beta} = \begin{cases} -\gamma \frac{\beta_0 - \hat{\alpha}}{(\beta - \alpha)^2}, & \text{if } \beta \geq \beta_0 \\ \gamma \frac{\hat{\alpha} - \alpha}{(\beta - \alpha)^2}, & \text{if } \beta < \beta_0 \end{cases} \quad (73)$$

On the other hand, according to (66) we can write the partial derivatives of G as:

$$\frac{\partial G}{\partial \beta} = Z(\beta) - 1 - \log Z(\beta) + (\beta - \alpha) \frac{\partial Z}{\partial \beta} \left[1 - \frac{1}{Z(\beta)} \right] \quad (74)$$

From (72) and (73) we have:

$$\frac{\partial G}{\partial \beta} = \begin{cases} f_1(Z), & \text{if } \beta \geq \beta_0 \\ f_2(Z, \beta), & \text{if } \beta < \beta_0 \end{cases} \quad (75)$$

where:

$$f_1(Z) = -\log(Z) + 1 - \frac{1}{Z}$$

$$f_2(Z, \beta) = Z - 1 - \log(Z) + \gamma \frac{\hat{\alpha} - \alpha}{\beta - \alpha} \left[1 - \frac{1}{Z} \right] \quad (76)$$

We can rewrite $f_1(Z)$ as:

$$f_1(Z) = \log \left(\frac{1}{Z} \right) + 1 - \frac{1}{Z} = - \left[\frac{1}{Z} - 1 - \log \left(\frac{1}{Z} \right) \right] \quad (77)$$

From (66), we know that $Z > 1 + \epsilon$ for some $\epsilon > 0$, and all β . We also know that $s(t) = t - 1 - \log(t)$ is always positive and only zero at $t = 1$. Hence, there is a constant $c > 0$ such that $f_1(Z) < -c$ for all $z \geq 1 + \epsilon$. So,

$$\frac{\partial G}{\partial \beta} < -c \text{ for } \beta > \beta_0. \quad (78)$$

For the region $\beta \in [\alpha^0, \beta^0]$, we can break $f_2(Z, \beta)$ into two parts, $f_2^1(Z)$ and $f_2^2(Z, \beta)$, where:

$$f_2^1(Z) = Z - 1 - \log(Z) \quad (79)$$

$$f_2^2(Z, \beta) = \gamma \frac{\hat{\alpha} - \alpha}{\beta - \alpha} \left[1 - \frac{1}{Z} \right]. \quad (80)$$

We have $f_2^2(Z, \beta) \geq 0$ since $\hat{\alpha} = \max\{\alpha_0, \alpha\}$ and $Z > 1$. Also, similarly to above, we can show that $f_2^1(Z) > c$ for some c and all $Z = Z(\alpha, \beta)$ with $\beta \in [\alpha^0, \beta^0]$. Therefore,

$$\frac{\partial G}{\partial \beta} > c \text{ for } \beta < \beta_0. \quad (81)$$

□

C. Proof of Theorem 4

To make the dependence on N explicit, let $\hat{a}_N^{(t)}$ and $\hat{b}_N^{(t)}$ be the outputs of the equivalent algorithm, Algorithm. 3 with the input size N . Let $\hat{\alpha}_N^{(t)}$ and $\hat{\beta}_N^{(t)}$ be the normalized values

$$\hat{\alpha}_N^{(t)} = \frac{\hat{a}_N^{(t)}}{N}, \quad \hat{\beta}_N^{(t)} = \frac{\hat{b}_N^{(t)}}{N}. \quad (82)$$

Since $\hat{a}_N^{(t)}$ and $\hat{b}_N^{(t)}$ are integer multiples of $\ell_t = N2^{-t}$, the normalized estimates $\hat{\alpha}^{(t)}$ and $\hat{\beta}^{(t)}$ will be on the discrete points:

$$\hat{\alpha}^{(t)} = \hat{i}^{(t)} 2^{-t}, \quad \hat{\beta}^{(t)} = \hat{j}^{(t)} 2^{-t}, \quad (83)$$

for some integer indices

$$\hat{i}^{(t)}, \hat{j}^{(t)} = 0, \dots, 2^t. \quad (84)$$

Now fix any $t_0 \geq 0$. Since there are at most a finite number of choices in (84), Lemma 6 shows that, with probability one, there exists an N_0 such that

$$|G_N(i2^{-t}, j2^{-t}) - G(i2^{-t}, j2^{-t})| < c2^{-t-2}, \quad (85)$$

where c is the constant in Lemma 8 and all the inequality is valid for all $i, j = 0, \dots, 2^t$, $N \geq N_0$ and $t \leq t_0$. We are now ready to prove our main induction step.

Lemma 9. Let t_0 and N_0 be defined as above. With probability one, for all $N \geq N_0$ and $t \leq t_0$, $\hat{\alpha}_N^{(t)}$ and $\hat{\beta}_N^{(t)}$ have a bounded

distance from the true values α^0 and β^0 in the sense that:

$$|\hat{\alpha}_N^{(t)} - \alpha^0| \leq \delta_t, \quad |\hat{\beta}_N^{(t)} - \beta^0| \leq \delta_t \quad (86)$$

where $\delta_t = 2^{-t}$.

Proof. We use induction for the proof. For $t = 0$ we will have:

$$|\hat{\alpha}^{(0)} - \alpha^0| \leq 1, \quad |\hat{\beta}^{(0)} - \beta^0| \leq 1 \quad (87)$$

since all values are in the interval $[0, 1]$ and thus their distance cannot be larger than 1. Suppose (86) holds for the t -th step with $t < t_0$. We prove that (86) also holds for step $t + 1$. The optimization over the left boundary in Algorithm. 3 is equivalent to

$$\hat{a}_N^{(t+1)} = \operatorname{argmax}_a J_N([a, \hat{b}_N^{(t)}]), \quad (88)$$

$$\text{s.t. } a = \hat{a}_N^{(t)} + \{0, \pm \ell_{t+1}\}. \quad (89)$$

Since $\ell_{t+1} = \delta_{t+1}N$, (82) and (61) show that the maximization can be re-written as:

$$\begin{aligned} \hat{\alpha}_N^{(t+1)} &= \operatorname{argmax}_{\alpha} G_N(\alpha, \hat{\beta}_N^{(t)}) \\ \text{s.t. } \hat{\alpha}_N^{(t+1)} &= \hat{\alpha}_N^{(t)} + \{0, \pm \delta_{t+1}\}. \end{aligned} \quad (90)$$

Now define

$$\begin{aligned} \alpha_N^{(t+1)} &= \operatorname{argmax}_{\alpha} G(\alpha, \hat{\beta}_N^{(t)}) \\ \text{s.t. } \alpha_N^{(t+1)} &= \hat{\alpha}_N^{(t)} + \{0, \pm \delta_{t+1}\}, \end{aligned} \quad (91)$$

which is identical to $\hat{\alpha}^{(t+1)}$, except that we have replaced $G_N(\cdot)$ in the objective function with $G(\cdot)$, its asymptotic limit. So, there are three possibilities for $\alpha_N^{(t+1)}$. We will show that

$$|\hat{\alpha}_N^{(t+1)} - \alpha^0| \leq \delta_{t+1} \quad (92)$$

for all three cases.

Case 1: $\alpha_N^{(t+1)} = \hat{\alpha}_N^{(t)}$. In this case, we know that

$$G(\hat{\alpha}_N^{(t)}, \hat{\beta}_N^{(t)}) \geq G(\hat{\alpha}_N^{(t)} - \delta_{t+1}, \hat{\beta}_N^{(t)}) \quad (93a)$$

$$G(\hat{\alpha}_N^{(t)}, \hat{\beta}_N^{(t)}) \geq G(\hat{\alpha}_N^{(t)} + \delta_{t+1}, \hat{\beta}_N^{(t)}) \quad (93b)$$

Since $G(\alpha, \hat{\beta}_N^{(t)})$ has a triangular maximum at $\alpha = \alpha^0$ it must be that α^0 is in the interval

$$\alpha^0 \in [\hat{\alpha}_N^{(t)} - \delta_{t+1}, \hat{\alpha}_N^{(t)} + \delta_{t+1}]. \quad (94)$$

Hence either:

$$\alpha^0 \in [\hat{\alpha}_N^{(t)} - \delta_{t+1}, \hat{\alpha}_N^{(t)}] \quad (95)$$

or

$$\alpha^0 \in [\hat{\alpha}_N^{(t)}, \hat{\alpha}_N^{(t)} + \delta_{t+1}]. \quad (96)$$

WLOG assume α^0 is in the interval (95). Therefore,

$$\begin{aligned} G_N(\hat{\alpha}_N^{(t+1)} + \delta_{t+1}, \hat{\beta}_N^{(t)}) &\stackrel{(a)}{<} G(\hat{\alpha}_N^{(t+1)} + \delta_{t+1}, \hat{\beta}_N^{(t)}) + \frac{c}{2}\delta_{t+1} \\ &\stackrel{(b)}{\leq} G(\hat{\alpha}_N^{(t+1)}, \hat{\beta}_N^{(t)}) + \frac{c}{2}\delta_{t+1} - c\delta_{t+1} \\ &\stackrel{(c)}{<} G_N(\hat{\alpha}_N^{(t+1)}, \hat{\beta}_N^{(t)}) - \frac{c}{2}\delta_{t+1} + \frac{c}{2}\delta_{t+1} \\ &\leq G_N(\hat{\alpha}_N^{(t+1)}, \hat{\beta}_N^{(t)}) \end{aligned} \quad (97)$$

where (a) is due to (85); (b) is due the fact that $G(\alpha, \beta)$ has a triangular maxima at $\alpha^0 \leq \hat{\alpha}_N^{(t)}$; and (c) again is due to (85). Hence in the maximization (90), we have that either $\hat{\alpha}^{(t+1)}$ satisfies

$$\hat{\alpha}^{(t+1)} \in \{\hat{\alpha}^{(t)} - \delta_{t+1}, \hat{\alpha}^{(t)}\}. \quad (98)$$

Since by assumption α^0 is in the interval (95), it follows that (92) holds.

Case 2: $\alpha_N^{(t+1)} = \hat{\alpha}_N^{(t)} + \delta_{t+1}$. In this case, we know that

$$G(\hat{\alpha}_N^{(t)} + \delta_{t+1}, \hat{\beta}_N^{(t)}) \geq G(\hat{\alpha}_N^{(t)} - \delta_{t+1}, \hat{\beta}_N^{(t)}) \quad (99a)$$

$$G(\hat{\alpha}_N^{(t)} + \delta_{t+1}, \hat{\beta}_N^{(t)}) \geq G(\hat{\alpha}_N^{(t)}, \hat{\beta}_N^{(t)}). \quad (99b)$$

Since $G(\alpha, \hat{\beta}_N^{(t)})$ has a triangular maximum at $\alpha = \alpha^0$, it must be $\alpha^0 \geq \hat{\alpha}_N^{(t)}$. Also, by the induction hypothesis, $\alpha^0 \leq \hat{\alpha}_N^{(t)} - \delta_t$. Hence, α^0 is in the interval:

$$\alpha^0 \in [\hat{\alpha}_N^{(t)}, \hat{\alpha}_N^{(t)} + \delta_t]. \quad (100)$$

In particular, since $\delta_{t+1} = \delta_t/2$, either:

$$\alpha^0 \in [\hat{\alpha}_N^{(t)}, \hat{\alpha}_N^{(t)} + \delta_{t+1}] \quad (101)$$

or

$$\alpha^0 \in [\hat{\alpha}_N^{(t)} + \delta_{t+1}, \hat{\alpha}_N^{(t)} + 2\delta_{t+1}]. \quad (102)$$

Suppose that α^0 is in the interval (101). Then, a similar argument as (97) shows that

$$G_N(\hat{\alpha}_N^{(t+1)} - \delta_{t+1}, \hat{\beta}_N^{(t)}) \leq G_N(\hat{\alpha}_N^{(t+1)}, \hat{\beta}_N^{(t)}) \quad (103)$$

which shows that $\hat{\alpha}_N^{(t)}$ must be one of the two values:

$$\hat{\alpha}^{(t+1)} \in \{\hat{\alpha}^{(t)}, \hat{\alpha}^{(t)} + \delta_{t+1}\}. \quad (104)$$

Since α^0 is in the interval (101) we see that (92) is satisfied. Similarly, if α^0 is in the interval (102), one can show that

$$G_N(\hat{\alpha}_N^{(t+1)}, \hat{\beta}_N^{(t)}) \leq G_N(\hat{\alpha}_N^{(t+1)} + \delta_{t+1}, \hat{\beta}_N^{(t)}), \quad (105)$$

which shows that $\hat{\alpha}_N^{(t)}$ must be one of the two values:

$$\hat{\alpha}^{(t+1)} \in \{\hat{\alpha}^{(t)} + \delta_{t+1}, \hat{\alpha}^{(t)} + 2\delta_{t+1}\}. \quad (106)$$

Since α^0 is in the interval (102) we see that (92) is satisfied.

Case 3: $\alpha_N^{(t+1)} = \hat{\alpha}_N^{(t)} + \delta_{t+1}$. Similarly to case 2, we can show that (92) is satisfied.

Hence, we have shown that, in all three cases (92) is satisfied. The proof for $|\hat{\beta}_N^{(t+1)} - \beta^0| \leq \delta_{t+1}$ is similar. So, by induction, (86) is satisfied for all $N \geq N_0$ and $t \leq t_0$ with probability one. $\square$

Now we proceed to prove (39). We have:

$$\text{IoU}_N^{(t)} = \frac{|\hat{S}_N^{(t)} \cap S_N^0|}{|\hat{S}_N^{(t)} \cup S_N^0|} \quad (107)$$

On the other hand, we have:

$$\begin{aligned} S_N^0 &= [a_N^0, b_N^0], \quad a_N^0 = \lfloor \alpha_0 N \rfloor, \quad b_N^0 = \lfloor \beta_0 N \rfloor, \\ \hat{S}_N^{(t)} &= [\hat{a}_N^{(t)}, \hat{b}_N^{(t)}], \quad a_N^{(t)} = \hat{a}_N^{(t)} N, \quad b_N^{(t)} = \hat{b}_N^{(t)} N \end{aligned} \quad (108)$$

So at the limit we'll have:

$$\lim_{N \rightarrow \infty} \text{IoU}_N^{(t)} = \lim_{N \rightarrow \infty} \frac{|\min(\hat{\beta}_N^{(t)}, \beta_0) - \max(\hat{\alpha}_N^{(t)}, \alpha_0)|}{|\max(\hat{\beta}_N^{(t)}, \beta_0) - \min(\hat{\alpha}_N^{(t)}, \alpha_0)|} \quad (109)$$

Lemma 9 and the triangle inequality shows:

$$\begin{aligned} \lim_{N \rightarrow \infty} \min\{|\hat{\beta}_N^{(t)}, \beta_0\} &\geq \beta^0 - |\hat{\beta}^{(t)} - \beta^0| \geq \beta^0 - 2^{-t} \\ \lim_{N \rightarrow \infty} \max\{|\hat{\beta}_N^{(t)}, \beta_0\} &\leq \beta^0 + |\hat{\beta}^{(t)} - \beta_0| \leq \beta^0 - 2^{-t} \\ \lim_{N \rightarrow \infty} \min\{|\hat{\alpha}_N^{(t)}, \alpha\} &\geq \beta^0 - |\hat{\alpha}^{(t)} - \alpha^0| \geq \beta^0 - 2^{-t} \\ \lim_{N \rightarrow \infty} \max\{|\hat{\alpha}_N^{(t)}, \alpha\} &\leq \alpha^0 + |\hat{\alpha}^{(t)} - \alpha| \leq \alpha^0 - 2^{-t}. \end{aligned}$$

Since the nominator and denominator of the IoU in (109) are positive, we can bound its limit as:

$$\begin{aligned} \lim_{N \rightarrow \infty} \text{IoU}_N^{(t)} &\geq \frac{|\beta_0 - \alpha_0| - 2^{-(t+1)}}{|\beta_0 - \alpha_0| + 2^{-(t+1)}} \\ &= 1 - \frac{2^{-(t+2)}}{|\beta_0 - \alpha_0| + 2^{-(t+1)}} \geq 1 - \frac{2^{-(t+2)}}{\beta_0 - \alpha_0}, \end{aligned} \quad (110)$$

where the limit is true almost surely for any fixed t . Since there are a countable number of values of t , it follows that

$$\lim_{N, t \rightarrow \infty} \text{IoU}_N^{(t)} = 1 \quad (111)$$

almost surely, which proves (4).

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